Typical event horizons in AdS/CFT

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Abstract

We consider the construction of local bulk operators in a black hole background dual to a pure state in conformal field theory. The properties of these operators in a microcanonical ensemble are studied. It has been argued in the literature that typical states in such an ensemble contain firewalls, or otherwise singular horizons. We argue this conclusion can be avoided with a proper definition of the interior operators.

1. Introduction

For black holes in asymptotically anti-de Sitter spacetime there are two natural choices of vacua compatible with the symmetries. One such vacuum is the analog of the Boulware vacuum [1]: positive frequency field modes far from the black hole, defined with respect to the timelike Killing vector, annihilate the vacuum state. Another natural choice is the analog of the Hartle–Hawking vacuum [2], where positive frequency is defined with respect to time translations of smooth global slices. This choice of vacuum gives rise to entanglement between the left and right asymptotic regions of the maximally extended AdS–Schwarzschild Penrose diagram and balanced thermal fluxes of ingoing and outgoing modes.

The canonical and microcanonical ensembles of such black holes were first studied by Hawking and Page [3]. They reached the important conclusion that for sufficiently large black holes, relative to the AdS radius of curvature, the ensembles are thermodynamically stable. In this limit, the horizon entropy of the black hole dominates versus thermal excitations of matter fields. It is this limit that is of interest in the present work.

More recently, Marolf–Polchinski (MP) [4] studied, using the microcanonical ensemble, the number operator for Kruskal-like modes, those natural from the viewpoint of a freely falling observer. They argued that in the holographic approach to quantum gravity, in a typical eigenstate of such normalizable modes, this number operator would always be of order 1. This then implies that typical black holes always have violations of general covariance near the horizon. In the following we re-examine this analysis and find that with an alternate construction of the interior observables, this conclusion can be avoided.

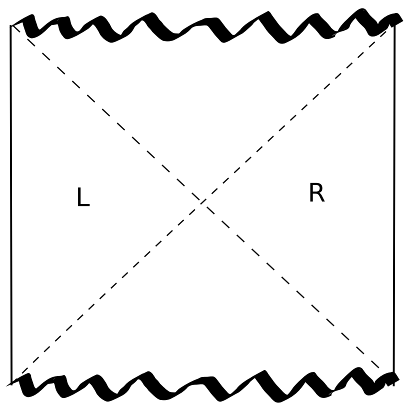

Figure 1: The Penrose diagram for Schwarzschild-anti de Sitter spacetime.

2. Semi-classical approach

Let us review the general setup. Following MP, we wish to consider the microcanonical ensemble of a CFT with a holographic asymptotically AdS description. The ensemble is defined by fixed mass M. We choose M sufficiently large that black holes give the dominant contribution to the entropy in the bulk. Thus, we expect the bulk description (up to exponentially suppressed corrections) to be a mass M asymptotically AdS black hole. Let us consider the effective field theory description of the bulk. As is well-known, in curved backgrounds there is no preferred mode decomposition of the field.

The b-modes are eigenstates of the timelike Killing vector at infinity, which is closely associated with the CFT Hamiltonian. The Penrose diagram for the anti-de Sitter Schwarzschild black hole is shown in figure 1. While the setup we are considering has a single CFT and a single asymptotic AdS spacetime, it is convenient to introduce the full Schwarzschild spacetime and use the left patch as a way to parametrize interior modes.

A bulk scalar field in the right patch (R) may be decomposed as

$$\phi = \sum_k b_{R,k}\phi_{R,k} + b^\dagger_{R,k}\phi^*_{R,k}$$

where k schematically represents the set of labels. We refer to these modes as the right b-modes, and they annihilate the Schwarzschild vacuum state $b_{R,k}|0\rangle_S = 0$.

If the CFT description is to include a description of the black hole interior, one must also consider a set of operators representing the fields in the left patch (L), which propagate into the upper patch in figure 1. Both sets of modes are needed to provide a complete description of the field in the upper patch. The full decomposition, valid in all coordinate patches is then

$$\phi = \sum_k b_{L,k}\phi_{L,k} + b^\dagger_L\phi^*_{L,k} + b_{R,k}\phi_{R,k} + b^\dagger_{R,k}\phi^*_{R,k}$$

and the Schwarzschild vacuum state is also annihilated by the left b-modes $b_{L,k}|0\rangle_S = 0$. Note that we define the mode functions, $\phi_{L/R}$, in the above such that they only have support in the appropriate (left/right) region.

One may also choose to decompose the field with respect to Kruskal modes, which are analytic across the horizon

$$\phi = \sum_k a_k \phi_{K,k} + a_k^\dagger \phi_{K,k}^* \,.$$

We refer to these modes as the a-modes. These modes annihilate the Kruskal/Hartle-Hawking vacuum $a_k|0\rangle_K = 0$. These modes look rather complicated when decomposed into frequencies with respect to the timelike Killing vector at infinity. However as shown in [5], these may be rewritten in terms of another set of operators $d_{L,k}$ and $d_{R,k}$ that annihilate $|0\rangle_K$ but are simply related to the b-modes

$$\begin{aligned}\phi &= \sum_k \left(2\sinh\left(\beta\omega_k/2\right)\right)^{-1/2} \Big(d_{R,k}\left(e^{\beta\omega_k/2}\phi_{R,k} + e^{-\beta\omega_k/2}\phi_{L,-k}^*\right) \\ &\quad + d_{L,k}\left(e^{-\beta\omega_k/2}\phi_{R,-k}^* + e^{\beta\omega_k/2}\phi_{L,k}\right)\Big) + h.c.\end{aligned}$$

where β is the inverse Hawking temperature of the black hole and ω_k is the positive frequency associated with the mode labeled by k. These operators are related to the b-mode operators by a Bogoliubov transformation

$$\begin{aligned}b_{L,k} &= \left(2\sinh\left(\beta\omega_k/2\right)\right)^{-1/2}\left(e^{\beta\omega_k/2}d_{L,k} + e^{-\beta\omega_k/2}d_{R,-k}^\dagger\right) \\ b_{R,k} &= \left(2\sinh\left(\beta\omega_k/2\right)\right)^{-1/2}\left(e^{\beta\omega_k/2}d_{R,k} + e^{-\beta\omega_k/2}d_{L,-k}^\dagger\right)\end{aligned}$$

which allows the different vacua to be related via

$$\begin{aligned}|0\rangle_K &= \prod_k \exp\left(e^{-\beta\omega_k/2}b_{L,k}^\dagger b_{R,k}^\dagger\right)|0\rangle_S \\ &= \prod_k \sum_{n_k=0}^{\infty} e^{-\beta\omega_k n_k/2}|N_{b,L,k} = n_k\rangle \times |N_{b,R,k} = n_k\rangle \qquad (1)\end{aligned}$$

where $N_{b,L,k}$ and $N_{b,R,k}$ are the number operators for the b-modes.

The number operator relevant for an infalling observer can be defined as the number operator built from the Kruskal mode number operators

$$N_{d,k} = d_{L,k}^\dagger d_{L,k} + d_{R,k}^\dagger d_{R,k} \qquad (2)$$

and this annihilates $|0\rangle_K$. In fact, everything we say applies to each term in the above separately. We will utilize this expression momentarily.

The MP argument instructs us to compute the ensemble average of the expectation value of $N_{d,k}$. Since the b-modes' frequencies are related to the CFT Hamiltonian, it is natural to evaluate the ensemble average in N_b eigenstates. It follows from the Bogolyubov transformation that each N_b eigenstate gives an $O(1)$ contribution to the expectation value of $N_{d,k}$. Since the number operator is positive definite there can be no cancellation in computing the ensemble average. Since this applies to each Kruskal-like mode, one concludes that the typical microstate of the ensemble has a firewall. We return to this argument later.

3. Euclidean quantum gravity approach

The Euclidean Gravity framework imposes periodicity in imaginary time to formulate the canonical ensemble [3]. The microcanonical ensemble is then defined via an inverse Laplace transform of the canonical ensemble. In the gravitational sector, the correct Bekenstein–Hawking black hole entropy is obtained. However in this approach the horizon entropy arises from geometric factors, rather than from state counting.

In addition, there is a contribution due to a thermal bulk field modes. This contribution can be viewed as computing the entropy of the reduced density matrix obtained by starting with the pure state (1) and tracing over the left-modes. For sufficiently large total energies, the microcanonical ensemble is dominated by exclusively the black hole entropy contribution, with a negligible term coming from thermal field configurations outside the black hole horizon. This observation will be important later, as it is necessary for a typical bulk mode to be in the global vacuum state (i.e. Hartle–Hawking vacuum) in order that a firewall not be seen.

4. AdS/CFT Approach

There is strong evidence the CFT is able to correctly reproduce the Bekenstein–Hawking contribution to the entropy in the large mass limit [6]. The entropy is reproduced by direct state counting, up to an overall constant that is difficult to determine precisely, because the CFT is strongly coupled in the limit that it is dual to a gravitational phase.

This approach must also yield significant corrections to the approach of section 2. Let us focus on the case of the four-dimensional bulk spacetime theory for the sake of definiteness. The boundary of the theory is $S^2 \times \mathbb{R}$, with the $\mathbb{R}$ factor corresponding to the time coordinate. Because the spatial sections are compact spheres, the energy spectrum of the conformal field theory becomes discrete. This induces a particular cutoff on the spectrum of the bulk theory.

Reconstructing bulk fields from CFT data

The reconstruction of bulk fields in the eternal black hole geometry was considered in [7, 8, 9]. There it is possible to build bulk fields in the interior of the horizon, but at the price of realizing the bulk fields as operators in a "doubled" CFT, corresponding to the two asymptotic infinities of the eternal black hole. To describe the microcanonical ensemble, one would like to solve the problem for black holes with a single asymptotic exterior region,corresponding to states in a "single" CFT. In this case the methods of [7] readily generalize only to the exterior of the black hole. Let us consider various possible hypotheses for realizing bulk operators in the black hole interior.

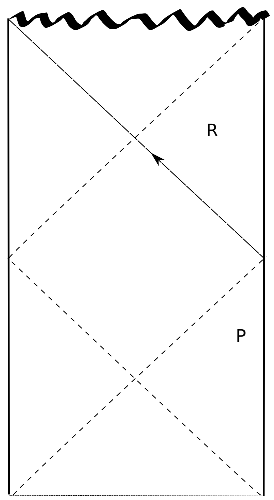

Figure 2: A black hole with a single exterior region. The upper dashed line shows the global horizon. The infalling matter is denoted by the line with the arrow.

4.0.1. No interior bulk operators

This is certainly a logical possibility in view of the semiclassical spacetime of figure 1. The exterior region exists for infinite time, and any CFT state should be mapped uniquely into some configuration of the exterior bulk fields (and perhaps more exotic configurations like strings). In particular, it should be a good approximation to map CFT operators into operators that produce some superposition of b-mode eigenstates. Since the exact spectrum is discrete, one can try to map any such CFT state into some superposition of only a finite number of b-mode eigenstates. This step is crucial for the diagonalization argument in MP. As pointed out in [10], there is no reason the discretization induced by the CFT will map to a finite number of b-mode eigenstates. In fact, the arguments of the next subsection suggest they at least start out being well-represented by superpositions involving infinite numbers of b-mode eigenstates as in (1). In essence, we claim that diagonalizing N_b and going to the microcanonical ensemble may not be compatible, in view of the need to cut off the effective field theory description in some way. Some may view this as a breakdown of the bulk effective field theory, and thus consistent with the MP argument; however, following Lowe and Thorlacius [11] we claim that the breakdown need not have severe consequences for reasonable observables.

4.0.2. Entangled bulk fields

The previous picture fails if one takes into account the recurrence of bulk field configurations due to the discreteness of the exact spectrum. For a quantum state, one expects a Poincare recurrence (i.e. for some set of observables to exhibit a quasi-periodicity) on some timescale ranging from the Heisenberg time e^S (at a minimum) to of order e^{ce^S} for some constant $c > 0$. If one combines

this with an ergodic hypothesis, then over such large timescales, one expects to observe time intervals when the horizon of the black hole disappears and then reforms due to collapse.

In this situation, as shown in figure 2, we can freely evolve fields in the region inside the global horizon back to the past region P. In this region, one can apply the construction of [7, 8, 9] to reproduce the bulk fields in terms of CFT operators. Thus we are free to time evolve forward, to obtain local bulk fields inside the horizon.

At first sight, it seems this should be sufficient to time evolve the bulk fields throughout the interior of the horizon. However this is limited by the following consideration. One only expects the exact theory to be able to reproduce local bulk fields on length scales larger than the Planck length (at best). However to time evolve bulk fields throughout the horizon interior, it is necessary to construct bulk fields on much shorter length scales in region P. The horizon redshift factor then expands these scales, when viewed from the perspective of late-time freely falling observers in the black hole interior. This limits the time after formation of the black hole at which this procedure will work to Schwarzschild times of order $\beta \log S$, where β is the inverse Hawking temperature and S is the Bekenstein-Hawking entropy. This timescale matches the scrambling time obtained using different methods in [12].

For pre-scrambling times after black hole formation, the interior bulk fields are correlated with the exterior bulk fields, and one expects to reproduce a good approximation to the results in the global vacuum, modulo excitations set up near the horizon prior to formation. From the CFT viewpoint, this means that the bulk operators cannot produce states that are superpositions of finite numbers of b-mode eigenstates—rather any state with these properties must be viewed as some superposition of AdS global mode eigenstates, each of which can only be represented as a superposition of an infinite number of b-mode eigenstates [10]. The arguments of MP do not hold in this situation.

The approach of [13] may be considered a variant on this theme. There one attempts to construct operators with the properties of interior operators out of the complex late-time exterior bulk operators. However this approach appears to fail consistency checks. There is no explanation of why an interior observer would experience quantum mechanics to a good approximation. The candidate local interior operators do not commute with non-local exterior operators at spacelike separations in the bulk. In the case of AdS/CFT, we believe the time-evolution of these states by the CFT Hamiltonian leads them to decohere into states that look local with respect to the usual bulk exterior Hamiltonian at sufficiently late times. The local interactions of the black hole with its exterior environment are of course what is responsible for this decoherence. By choosing a sufficiently extreme exterior environment, this decoherence time may be made quite short [11], of order a scrambling time. By the same token, this local exterior bulk time evolution will then look highly non-local from the viewpoint of these candidate interior operators. So while the construction of such operators may appear to work at the level of free field theory, it is not expected to survive the inclusion of interactions.

4.0.3. Unentangled bulk fields

An extension of the idea of section 4.0.1 is to assume in addition to the exterior bulk fields, one also has some extra labels that represent the interior state of the bulk fields. In view of the arguments of section 3 this might seem reasonable, as one expects some set of extra labels to account for the horizon entropy. However this runs counter to the ideas of black hole complementarity, where one expects some kind of different complementary description of the interior states, and that only the exterior theory need have a conventional unitary description.

In any case, this is the set of assumptions presented in MP, and if one uses the resulting interior operators to build number operators for infalling modes, one soon finds highly excited states in any typical state in the microcanonical ensemble.

The two-sided black hole of figure 1 may be viewed as a particular example of this approach. The CFT factors into two disjoint products $CFT_L \times CFT_R$. The construction of local bulk interior operators in this framework has been described in [8, 9, 7]. The microcanonical ensemble might be defined not just counting states of fixed energy in CFT_R, taking the right region to represent the black hole exterior, but also taking arbitrary energy in CFT_L or any distribution thereof. Because the propagation of states into the interior involve potentially arbitrary states propagating from the left asymptotic region, the future interior region will typically contain a firewall.

4.0.4. Ancillae

This approach may be viewed as an extension of the approach of section 4.0.2 to define interior bulk operators at arbitrary times after black hole formation. This procedure is described in detail in [11]. The basic elements are the exact state in the CFT, mapped to some exterior bulk state on a family of timeslices centered on some infalling geodesic at horizon crossing. By doing propagation back of order a scrambling time $\beta \log S$, combined with the introduction of additional modes in an entangled pure state (commonly called ancillae in the quantum computing literature), which describe sub-Planck length modes in their vacuum state. Such a construction is known as an isometry, because the norm of the states is preserved, despite the dimension of the Hilbert space being enlarged. One can propagate this state forward in time, back to the horizon crossing time and beyond, to construct the quantum state and associated local operators in a complementary theory relevant for an infalling observer falling into the interior.

In this picture, the modes with wavelengths shorter than the temperature scale β (at the horizon crossing time and beyond) will arise directly from the vacuum ancillae modes. Thus no firewall is expected.

5. Discussion and conclusions

The approach of 4.0.1 is the minimal interpretation of the CFT data. As described in 4.0.2 it fails for tiny fractions of the time, for a large AdS black

hole. But nevertheless is the simplest interpretation that works at typical times. Let is consider the microcanonical ensemble of such black hole states. Certainly there is not a firewall in the exterior by the arguments of [14]. Local operators outside the horizon, in a typical pure state will see deviations from the purely thermal global vacuum result at order e^{-S}. By construction, one is prevented from asking the question whether there is a firewall in the interior.

In the approach of 4.0.3 one is able to build interior bulk operators. However since the construction then produces a firewall on the horizon, it is not self-consistent, since the construction of the bulk operators presumed a smooth semiclassical geometry around which to build operators describing small perturbations. Perhaps one should then conclude that rather than proving the existence of firewalls for typical states, that one's construction of interior operators has failed.

Of the approaches considered here, the only one that appears to be self-consistent is that of section 4.0.4 which produces no firewall at typical times and for a typical state in the microcanonical ensemble.

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